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Non-spherical collapse in AdS and Early Thermalization in RHIC

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Abstract: In the flat space, it is well known that non-spherical shell collapses to give globular cluster after many oscillations. However, we show that in anti de sitter space, arbitrary shape of shell or cloud falls to form a black hole in one dynamical time. By gague/gravity dualuty, this explains the early thermalization in strong quark-gluon plasma. This is traced back to the a remarkable property of AdS : the period in radial motion is the amplitude independent of the amplitude in spite of the NON-linearity of the equation of motion. We investigated the interaction effect numerically and observed the same qualitative behavior for the attractive forces. For repulsive interactions, particles halt in the sky for long time due to the specific structure of the propagator. It may be responsible for the hair creation in the AdS black hole.

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1 Introduction

One of the mystery in RHIC experiment is the early thermalization[1]. The fireball made in the collision seems to reach the equilibrium in 1fm/c, which is comparable time for gold ions to pass each other. This is attributed to the strongly interacting nature of the system. Then there is no way to understand this phenomena in perturbative field theory context, so one naturally relies on a dual formulation where particles are weakly coupled. According to the gauge/gravity duality [2–4], thermalization is dual to the black hole formation in Anti de Sitter(AdS) space. Our question in this formulation becomes "Why generic gravitational collapse produces black hole in one dynamical time in AdS space?"

For the collapse of a spherical shell, it is less surprising that the final result is the black hole. However, for non-spherical shell, the result is far from a black hole in flat space; there, a 'globular clusters' is formed rather than a black hole. Recent studies on scalar field collapse in AdS space [5–9] shows that even for the spherical symmetric collapse, black hole is made only after several reflection from the boundary. Therefore such result have some distance from what is happening in 'thermalization in one passing time'. The natural explanation of the experiment would be that the black hole is formed for generic initial configuration and also it should be in one dynamical time, that is, in one falling time.

Also, showing the black hole formation by numerical analysis does not give an 'understanding' why it is formed so easily in AdS but not in flat space. While some analytic discussion is provided in [6], it is not very clear to us what is the basic physical picture underlying such inevitability of black hole formation even for the spherical collapse. Confining or Box nature of AdS space does NOT explain the easy formation of black hole.

In this paper, we will study the collapse of non-spherical shell, and show that it forms a black hole and we also claim that we understand the essential difference between the AdS and flat space. We treat the shell as a collection of particles. This is very different from the scalar field collapse where it is assumed that matter is in a state of coherent condensation.

Only when particle's wave functions are overlapping, one can justify treating many particles in terms of condensation wave function or the scalar field configuration.

In the gauge/gravity duality one considers the limit where string is much smaller than the AdS radius. In this limit the wave nature as well as the stringy nature is suppressed. For the many particle system in RHIC, a scalar field configuration is not the proper dual configuration to the fireball. Therefore we consider the shell as a collection of the interacting particles in AdS. We will show that particles arrive at the center simultaneously regardless of their masses, initial positions and velocities. This means that arbitrary shape of shell will gradually becomes the spherical shell, as it falls. After such shell pass the 'would be horizon', black hole forms and particle's motion can not continue. We will first show this for particles without inter-particle interaction and then we will show that the same thing is true even in the presence of the interacting. In gauge/gravity duality, the dynamics of gluon exchange is treated by gravity background in leading 1/N expansion. So here neglecting inter-particle interaction means treating particle interaction in leading 1/N expansion of gauge theory.

Even in the simplest case of free fall along the radial direction, the equation of the motion (EOM) is that of a non-linear oscillator. Nevertheless its period is independent of the amplitude as if it is a simple harmonic oscillator(SHO). This is a remarkable property of AdS spacetime. We will de-mystify this phenomena by finding a non-linear mapping that transform the EOM of the falling into that of SHO. We call this property as synchronization effect of AdS. This means that any non-spherical shell in AdS space becomes more spherical shell as it falls, and all parts of the shell reach the center simultaneously. It means any cloud falls to become a black hole in AdS space. We will see that adding initial velocity does not change the conclusion. We believe that this property is the fundamental reason, from the dual point of view, behind the early thermalization of the strongly interacting quark gluon plasma.

2 Collapse in global AdS$_5$ without inter-particle interaction

To consider the collapse of non-spherical shell which is consist of non-interacting particles, we need to look at the motion of individual particles. For AdS_5 with spherical boundary the metric is given by

$$ds^2 = -(1 + r^2/R^2)c^2dt^2 + r^2d\Omega^2 + \frac{dr^2}{1 + r^2/R^2} \tag{2.1}$$

It is well known that the falling time from the boundary to the center following the null geodesic is $\pi R/2c$. However we will see that even for the massive particle starting from arbitrary position with zero radial velocity will arrive at the center after the same time. Notice that the time for the light starting from the arbitrary position to arrive at the center will not be the same.

The equation of motion is given by the action

$$S = -m \int \sqrt{-g_{\mu\nu}\dot{x^\mu}\dot{x^\nu}}dt, \text{ with } \dot{x} = \frac{dx}{dt}. \tag{2.2}$$

For simplicity, we first consider the radial motion and set $R = 1, c = 1$. The energy conservation can be written as

$$\frac{m(1+r^2)}{\sqrt{1+r^2-\dot{r}^2/(1+r^2)}} = E. \tag{2.3}$$

It is easy to see that the system describe an non-linear oscillator. If we assume that the particle starts with zero radial velocity from the initial radial position r_0, then

$$E = m\sqrt{1+r_0^2}, \tag{2.4}$$

which establish an dictionary between the total energy and the initial radial coordinate. Introducing v_c by $v_c = r_0/\sqrt{1+r_0^2}$, we have $E = \frac{m}{\sqrt{1-v_c^2}}$. Its velocity in the radial direction start with 0 and grows up to v_c when it reaches at the center. Interestingly, we can find the exact solution of the equation of motion:

$$r = \frac{v_c \cos t}{\sqrt{1-v_c^2\cos^2 t}}. \tag{2.5}$$

The remarkable property of this solution is that the period of the motion is 2π, independent of the original position r_0, as if it is an simple harmonic oscillator. Restoring the scale parameters R, c by $r, t, v \to r/R, t/R, v/c$ the falling time is

$$T_{fall} = \frac{\pi}{2}\frac{R}{c}. \tag{2.6}$$

This means that arbitrary set of particles, falling in AdS will form a black hole regardless of the initial position, provided, (i) all the particles start with zero initial velocities, and (ii) Interaction between the particles are negligible. See figure 1.

How do we justify or relax the conditions listed above?

1. In AdS/CFT correspondence, the radial direction is the dual of the energy scale. If two gold ions collided in RHIC and created particles of energy E_i and mass m_i, $i = 1, \cdots, N$ inside the fireball, then the holographic image of such fireball is the particles in AdS space at the position

   $$r_{0i} = \sqrt{(E_i/m_i)^2 - 1}. \tag{2.7}$$

   Particles created in the fireball have velocities in x^i directions but not in radial direction. Therefore their dual image particles do not either. This justifies the first issue listed above.

2. In gauge/gravity dual, the gluon dynamics is replaced by the gravitational background and the leading order inter-particle gluonic interaction in the many body quark-gluon plasma is approximated as the particle motions in a fixed gravitational background. Therefore interparticle interaction in AdS is necessary only to take care of the non-gluon interaction or non-leading order interaction in $\mathcal{O}(1/N_c)$, therefore such inter particle interaction in AdS bulk should be absent or very weak in the large N_c theory.

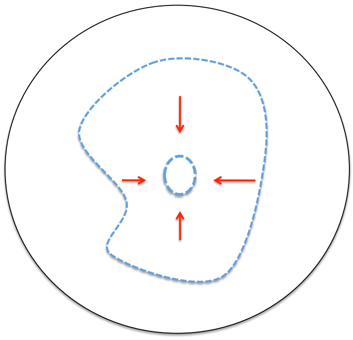

Figure 1. Collapse of a shell with arbitrary shape in AdS space. The whole shell will reach at the center simultaneously. $T_{fall} = \frac{\pi}{2}\frac{R}{c}$.

To de-mystify the amplitude independence of the period of the above nonlinear oscillators, we can actually map the above non-linear oscillator equation to that of the harmonic oscillator. If we define η by

$$r = \frac{\eta}{\sqrt{1-\eta^2}}, \tag{2.8}$$

the eq.(3.1) can be mapped to the harmonic oscillator;

$$\dot{\eta}^2 + \eta^2 = \eta_0^2 := 1 - \left(\frac{m}{E}\right)^2, \tag{2.9}$$

whose solution is $\eta = \eta_0 \cos t$. In this coordinate, the metric becomes

$$ds^2 = \frac{1}{1-\eta^2}\left(-dt^2 + \eta^2 d\Omega^2\right) + \left(\frac{d\eta}{1-\eta^2}\right)^2. \tag{2.10}$$

We can also define $\eta = \tanh\rho$ to get $r = \sinh\rho$ to get the metric in the global corordinate:

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega^2. \tag{2.11}$$

The point is that once the particles pass the would-be horizon of the system, there is no turning back. Apparent horizon formation is inevitable consequence of the synchronized falling.

3 Flat boundary case

When we replace the metric $g_{00} = 1 + r^2$ by $g_{00} = r^2$, we get the flat boundary AdS metric in Poincare patch where most of the of gauge/duality has been discussed. The integrated equation of motion is

$$\frac{mr^2}{\sqrt{r^2 - \dot{r}^2/r^2}} = E, \tag{3.1}$$

and its solution is [11]

$$r = \frac{\epsilon}{\sqrt{1+(\epsilon t/2)^2}}. \tag{3.2}$$

This is not a periodic solution. It show that starting from $r_0 = E/m = \epsilon$, it takes infinite time to reach at the center. However, the large time behavior is $r \sim 2/t$, independent of the initial height $r_0 = \epsilon$. This is a manifestation of the synchronization effect in this metric: two particles with different initial height falls and get closer, which is enough to argue the canonical formation of the black hole in this metric. This effect was observed in [11] as a 'tendency' of time focusing but due to the infinite falling time in Poincare coordinate time, it was not recognized there that synchronization is the exact property of the AdS space. Later, we will show that introducing the interaction does not change the situation.

3.1 Effect of the initial velocity

Here we study the effect of the initial velocity along the space time direction. Suppose the particle is in motion along x direction. Then two first integrals are

$$\frac{mr^2}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = E, \frac{mr^2\dot{x}}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = p, \tag{3.3}$$

which can be called as the energy and momentum respectively. If we set $V = p/E$,

$$r = \frac{\epsilon(1-V^2)}{\sqrt{1+(\epsilon(1-V^2)t/2)^2}}. \tag{3.4}$$

we get $\dot{x} = V$. Remarkably the large time behavior of the radial position is independent of all of the initial conditions m, E, p. Therefore we can say that the time focusing effect is perfect even in the presence of the motion along the collisional direction.

4 The effect of the Interaction

So far, we discussed the particles falling without interaction. We now discuss the effect of it in Poincare coordinate where the metric is

$$ds^2 = \frac{1}{x_0^2}(dx_0^2 + dx^\mu dx_\mu), \text{ with } x_0 = 1/r \tag{4.1}$$

The scalar propagator for particle with mass $m^2 = \Delta(\Delta - d)$ in the AdS$_{d+1}$ is given in [12] and it is given by

$$G \sim \left(\frac{1}{u(2+u)}\right)^\Delta \text{ with } u = \frac{(x-y)^M(x-y)_M}{2x_0y_0} \tag{4.2}$$

Since u is nonnegative and we are interested in the most singular contribution, we will neglect the factor $u+2$. The Newtonian potential in AdS can be derived from this to give

$$V(\{x_i, y_i\}) = \int\int d^5x d^5y J(x)G(x,y)J(y), \tag{4.3}$$

$$= G_N \sum_{i<j} \int dt \frac{(x_{i0}x_{j0})^{2\Delta}}{\left(|x_i(t)-x_j(t)|^2 + |x_{i0}(t)-x_{j0}(t)|^2\right)^{\Delta-1/2}} \tag{4.4}$$

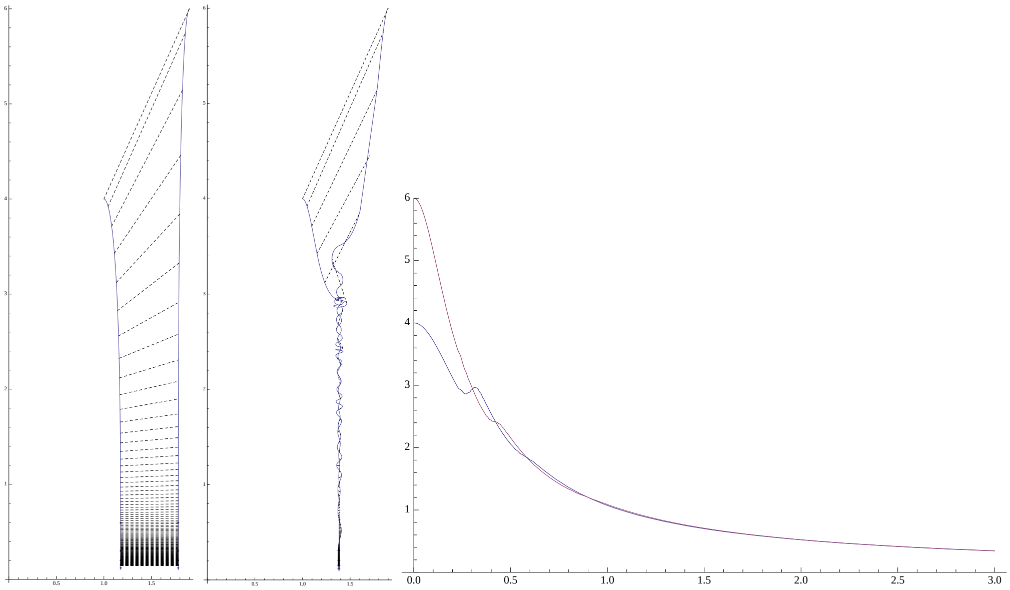

Figure 2. Falling in AdS with and initial velocity. : (Left) without inter-particle interaction. This is the flat-boundary analogue of Figure 1. (Middle) with interaction. r v.s x, x-axis is one of the space direction. (Right) radius as a funcntion of time. The synchronization effect (equalizing the radial position) is manifest here.

where $J(x) = \sum_i \delta^4(x^A - x_i^A(t)), A = 0, 1, 2, 3$. Since the initial velocities are only along the x^μ direction, we expect that the attractive interaction will only enhances the focusing effect in radial motion. Indeed we can verify this numerically, using the equation of motion derived from the Lagrangian

$$S = -m \int \sqrt{-g_{\mu\nu} \dot{x^\mu} \dot{x^\nu}} dt, \ +V(\{x_i, y_i\}) \tag{4.5}$$

In the numerical calculation we used $G_N = 1, \Delta = 3/2$ for simplicity.

In figure 2, falling of a few particles with some horizontal initial velocities starting from different heights are drawn. The calculation is done by the mathematica. The vertical lines are the falling trajectories and the horizontal dashed lines indicate the equal time slices. As time goes on, it is manifest that the radial positions converge. In figure 3, we consider what happens if inter-particle interactions are repulsive. This is possible if particles carry extra charges. Interestingly, we see that falling is halted for some moments at the certain radial region and then proceed to the black hole formation. We can understand such behavior from the structure of the propagator (4.2) : In the deep IR region, $u \to 0$ for any finitely distant two points, therefore strong repulsion is effective there although particles are separated enough. We speculate that this can be the mechanism of the formation of the gravitational hair which is observed in the theory of holographic superconductor.

In figure 4, we show what happens if three particles collide with inter-particle attraction and repulsion. As we can see, whatever is the situation, radial positions of particles converges. Therefore we conclude that such synchronization effect is not destroyed by the interaction effect, especially if the interaction is attractive. For the repulsive case, particles

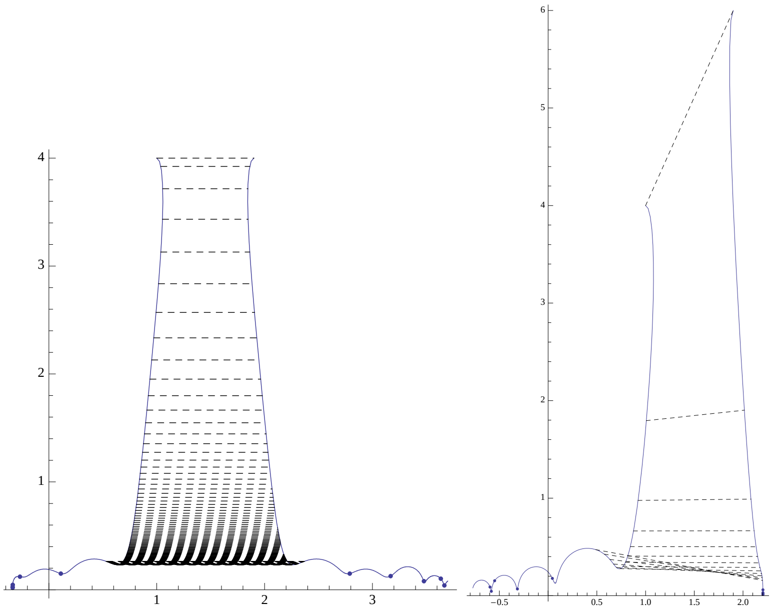

Figure 3. r v.s x. Falling in AdS with repulsive interaction. L) start from the same height. R) start from the different height.

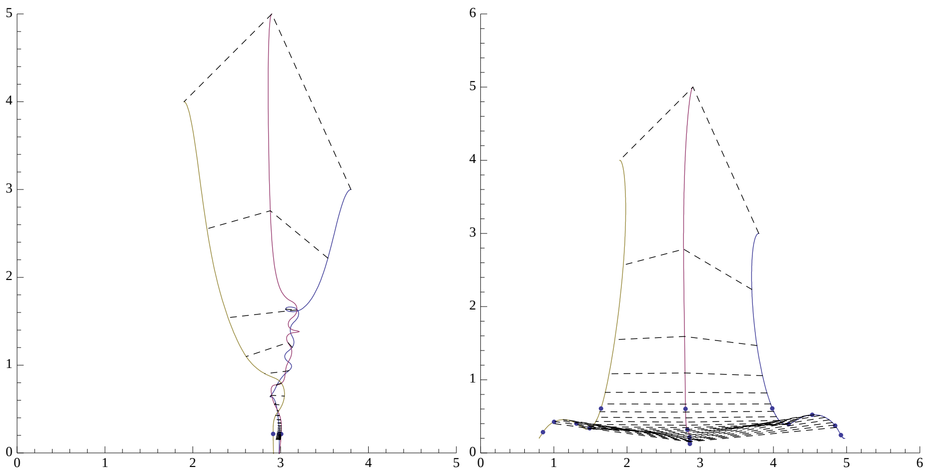

Figure 4. Three particles colliding in 5 dimension. L) with attractive interaction. R) with repulsion. All the particle arrive at the center simultaneously even in the presence of the interaction.

halts at the sky for long time due to the specific structure of the propagator: namely, $x_0 y_0$ factor in $1/u$. See (4.2) and (4.4). We believe that this is responsible for the presence of the hair in the AdS space.

5 Conclusion

In this paper we demonstrated that arbitrary shape of shell in AdS falls and form a black hole. The physical mechanism is the synchronized falling which is the characteristic property of AdS. While individual particle's motion is oscillation, many particle's motion should

be terminated by the black hole formation. Once the black hole is formed, there will be no more oscillation. The details of the stabilization of the system through losing the potential energy should be worked out by considering the back reaction of the metric, which is not the scope of this work.

The reason for several oscillation before black hole formation in the study of the spherical scalar field collapse is due to the wave nature of the scalar field: as the field configuration collapses, uncertainty principle activates the kinetic term [10], which generate pressure and causes the bounce. It can be also attributed to the effective repulsion between the scalar particles as we have seen in the section 4.

The future work is to find the effect of the extra interactions and to find the time dependent solution of Einstein equation that describe the formation of the black hole. Large time approximate solution was proposed in [14] and further developed in [15, 16]. However, finding exact Einstein gravity solution is non-trivial even for the spherically symmetric shell and we will postpone it to future work.

Acknowledgments

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References

[1] U. W. Heinz and P. F. Kolb, Nucl. Phys. A **702**, 269 (2002) [hep-ph/0111075].

[2] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv.Theor.Math.Phys.* **2** (1998) 231–252, [hep-th/9711200].

[3] S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys.Lett.* **B428** (1998) 105–114, [hep-th/9802109].

[4] E. Witten, *Anti-de Sitter space and holography*, *Adv.Theor.Math.Phys.* **2** (1998) 253–291, [hep-th/9802150].

[5] S. Bhattacharyya and S. Minwalla, JHEP **0909**, 034 (2009) [arXiv:0904.0464 [hep-th]].

[6] P. Bizon and A. Rostworowski, Phys. Rev. Lett. **107**, 031102 (2011) [arXiv:1104.3702 [gr-qc]].

[7] D. Garfinkle and L. A. Pando Zayas, Phys. Rev. D **84**, 066006 (2011) [arXiv:1106.2339 [hep-th]].

[8] D. Garfinkle, L. A. Pando Zayas and D. Reichmann, JHEP **1202**, 119 (2012) [arXiv:1110.5823 [hep-th]].

[9] A. Buchel, L. Lehner and S. L. Liebling, Phys. Rev. D **86**, 123011 (2012) [arXiv:1210.0890 [gr-qc]].

[10] S. -J. Sin, Phys. Rev. D **50**, 3650 (1994) [hep-ph/9205208].

[11] E. Shuryak, S. -J. Sin and I. Zahed, J. Korean Phys. Soc. **50**, 384 (2007) [hep-th/0511199].

[12] E. D'Hoker and D. Z. Freedman, Nucl. Phys. B **550**, 261 (1999) [hep-th/9811257].

[13] E. D'Hoker, D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli, Nucl. Phys. B **562**, 330 (1999) [hep-th/9902042].

[14] R. A. Janik and R. B. Peschanski, Phys. Rev. D **73**, 045013 (2006) [hep-th/0512162].

[15] S. Nakamura and S. -J. Sin, JHEP **0609**, 020 (2006) [hep-th/0607123].

[16] S. -J. Sin, S. Nakamura and S. P. Kim, JHEP **0612**, 075 (2006) [hep-th/0610113].